Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Tuesday 20 June 2023

Afternoon

Paper reference **9MA0/32**

Mathematics

Advanced
PAPER 32: Mechanics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50. There are 6 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

P 7 2 8 2 0 A 0 1 2 0

Pearson

Initial at rest suggests the initial velocity to be zero ($u=0$)

1. A car is initially at rest on a straight horizontal road.

The car then accelerates along the road with a constant acceleration of 3.2 m s^{-2}

Find

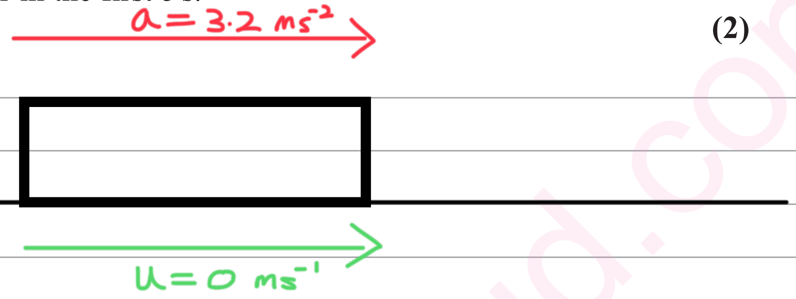
(a) the speed of the car after 5 s,

(1)

(b) the distance travelled by the car in the first 5 s.

(2)

1a



①

Initial velocity of the car $= 0$
Acceleration of the car $= 3.2$
Time of the car drived $= 5$
Final velocity of the car $= v$

we first identify the known and the unknown variable that we need

② Now we have v u a t then we can apply suvat equation

$V = u + at$

$V = 0 + 3.2 \times 5$

$V = 16$

1b

①

Initial velocity of the car $= 0$
Acceleration of the car $= 3.2$
Time of the car drived $= 5$
Final velocity of the car $= 16$
Displacement of the car for $5s = S$

we first identify the known and the unknown variable that we need

② We can use any suvat equation that include the displacement

$S = \frac{1}{2}(u+v) \times t$

$S = \frac{1}{2}(16) \times 5$

$S = 40$

Question 1 continued

(Total for Question 1 is 3 marks)

2.

Figure 1

A particle P has mass 5 kg.

The particle is pulled along a rough horizontal plane by a horizontal force of magnitude 28 N.

The only resistance to motion is a frictional force of magnitude F newtons, as shown in Figure 1.

(a) Find the magnitude of the normal reaction of the plane on P **(1)**

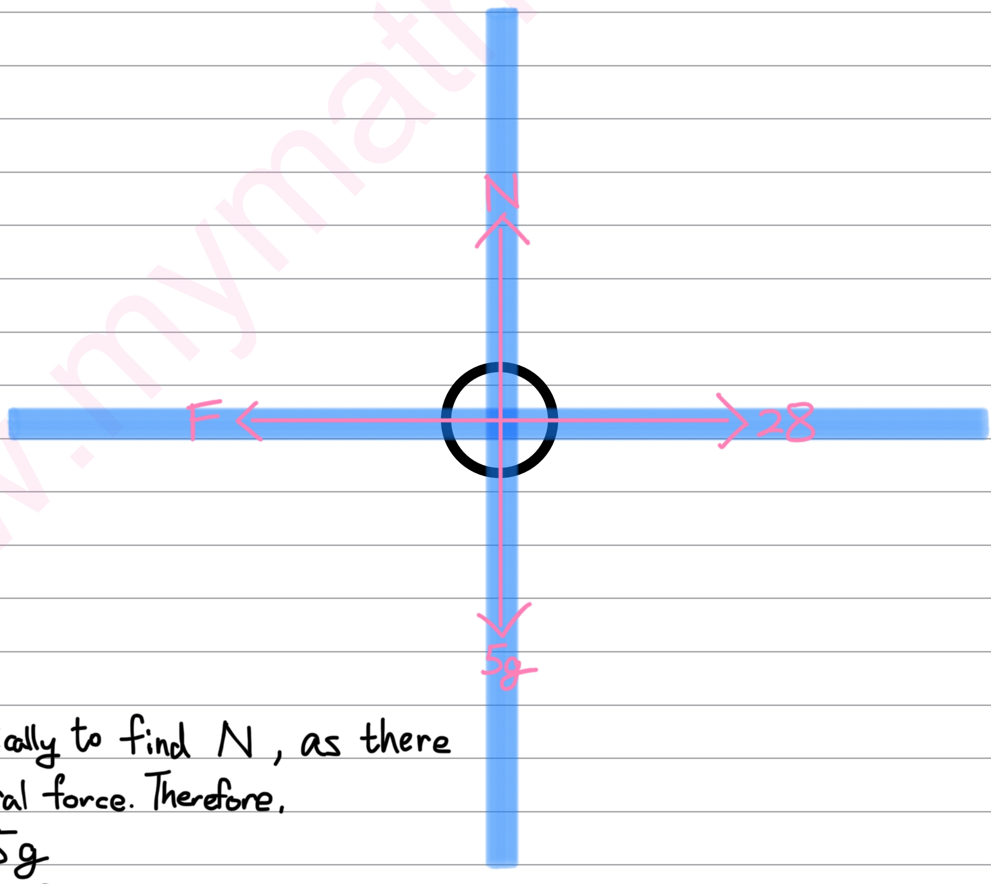
The particle is accelerating along the plane at 1.4 m s^{-2}

(b) Find the value of F **(2)**

The coefficient of friction between P and the plane is μ

(c) Find the value of μ, giving your answer to 2 significant figures. **(1)**

2a



Resolved Vertically to find N, as there no other vertical force. Therefore,

$N = 5g$

$N = 5 \times 9.8$

$N = 49$

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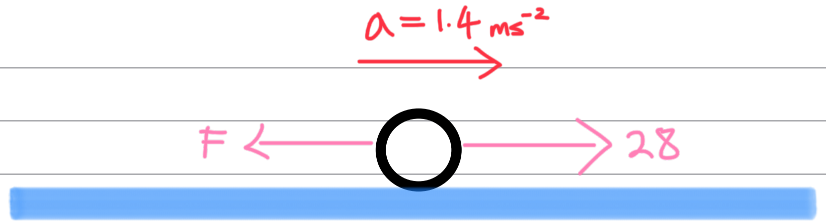
Question 2 continued

2b



We will need to resolve horizontally because force F is a horizontal force. Since the ball is moving to the right, the difference between 28 and F is the force that accelerate the ball to the right.

$28 - F = 5 \times 1.4$ ⇒ $F = ma$

$-F = 7 - 28$

$-F = -21$

$F = 21$ N

5 kg

acceleration to right is 1.4 ms⁻²

The difference between F and 28

2c. To caculate μ, we need to use the equation $F = \mu R$. F is 21 N (caculated in part b) and R is 49 N (caculated in part a)

$F = \mu R$

$21 = 49\mu$

$\frac{21}{49} = \mu$

$\mu = 0.426$

(Total for Question 2 is 4 marks)

3. At time t seconds, where $t \geqslant 0$, a particle P has velocity $\mathbf{v}\,\text{m}\,\text{s}^{-1}$ where

$$\mathbf{v} = (t^2 - 3t + 7)\mathbf{i} + (2t^2 - 3)\mathbf{j}$$

Find

(a) the speed of P at time $t = 0$ **(3)**

(b) the value of t when P is moving parallel to $(\mathbf{i} + \mathbf{j})$ **(2)**

(c) the acceleration of P at time t seconds **(2)**

(d) the value of t when the direction of the acceleration of P is perpendicular to $\mathbf{i}$ **(2)**

3a To find the velocity of P at time =0, we need to substitute t=0 into the equation velocity.

$v(t) = (t^2-3t+7)i + (2t^2-3)j$

$v(0) = (0^2-3\times0+7)i + (2\times0^2-3)j$

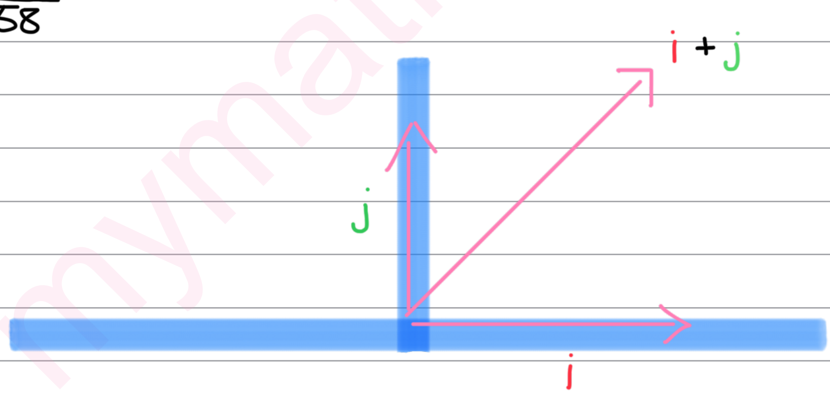
$v(0) = 7i - 3j$

Speed is the magnitude of velocity

$\text{Speed} = \sqrt{7^2+(-3)^2}$

$= \sqrt{58}$

3b



Since P is moving in the direction of (i + j). This means the i component of p will be equal to the j component because they are in 1:1 ratio.

$\frac{(t^2-3t+7)i}{(2t^2-3)j} = \frac{i}{j}$

$\frac{t^2-3t+7}{2t^2-3} = \frac{1}{1}$

$t^2-3t+7 = 2t^2-3$

$-t^2-3t+10 = 0$

$t = 2$ or -5 (rejected because time can't be -ve)

Question 3 continued

3c

$$\begin{tikzcd} S \arrow[r, bend left, "\text{Differentiation}"] & V \arrow[l, bend left, "\text{Integration}"] \arrow[r, bend left, "\text{Differentiation}"] & a \arrow[l, bend left, "\text{Intergration}"] \end{tikzcd}$$

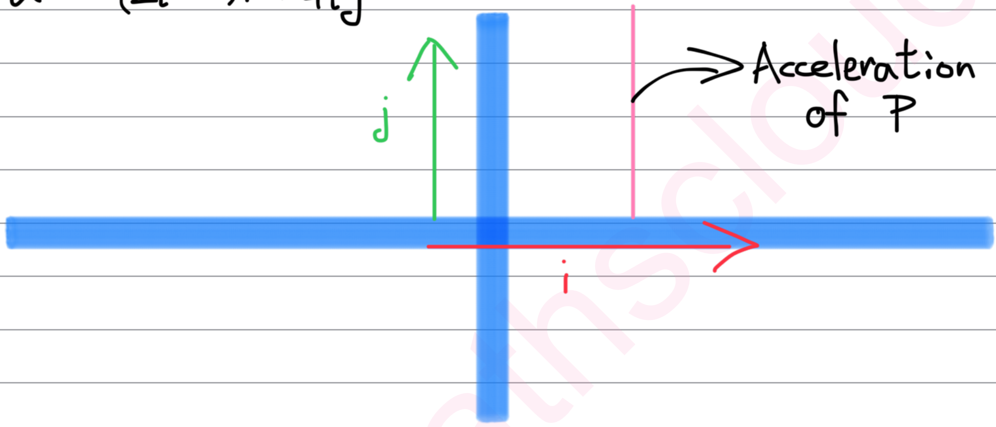
Acceleration is the rate of change of velocity so we can use differentiation to find the acceleration.

$$v = (t^2 - 3t + 7)i + (2t^2 - 3)j$$

$$\frac{dv}{dt} = (2t - 3)i + (4t)j$$

$$a = (2t - 3)i + 4tj$$

3d



As the acceleration of P is perpendicular to i component. This means that acceleration of P only have j component and NO i component

$$2t - 3 = 0$$

$$2t = 3$$

$$t = 1.5\ s$$

(Total for Question 3 is 9 marks)

4. [*In this question,* **i** *and* **j** *are horizontal unit vectors and position vectors are given relative to a fixed origin O*]

A particle P is moving on a smooth horizontal plane.

The particle has constant acceleration $(2.4\mathbf{i} + \mathbf{j})\,\mathrm{m\,s^{-2}}$

At time $t = 0$, P passes through the point A.

At time $t = 5\,\mathrm{s}$, P passes through the point B.

The velocity of P as it passes through A is $(-16\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-1}}$

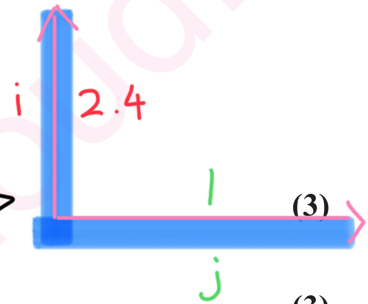
(a) Find the speed of P as it passes through B.

(4)

The position vector of A is $(44\mathbf{i} - 10\mathbf{j})\,\mathrm{m}$.

At time $t = T$ seconds, where $T > 5$, P passes through the point C.

The position vector of C is $(4\mathbf{i} + c\mathbf{j})\,\mathrm{m}$.



(b) Find the value of T.

(3)

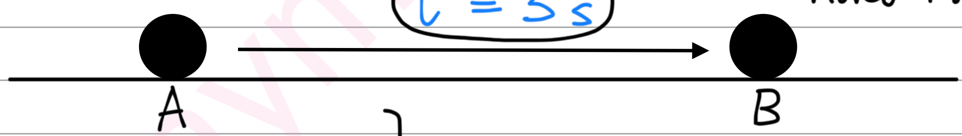
(c) Find the value of c.

(3)

4a

$a = 2.4i + j$

$t = 5s$ Total time spent for P Moved from A to B



① Initial velocity of P = $-16i - 3j$
Acceleration of P = $2.4i + j$
Time of P spent moved from A to B = 5
Final velocity of P = v

we first identify the known and the unknown variable that we need

② We can then apply suvat equation to find v

$V = u + at$

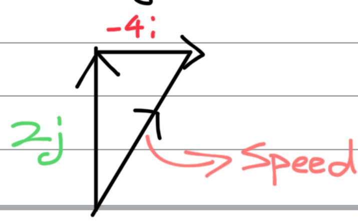
$V = -16i - 3j + 5(2.4i + j)$

$V = -4i + 2j$

③ Speed is the magnitude of velocity so we can use Pyth theorem to find speed

$\text{Speed} = \sqrt{(-4)^2 + 2^2}$

$= 2\sqrt{5}$



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Question 4 continued

4b ①

Initial velocity of $P = -16i - 3j$

Acceleration of $P = 2.4i + j$

Time of P spent moved from A to B = T

Displacement of P moved $= (4i + cj) - (44i - 10j)$

$= -40i + (c + 10j)$

we first identify the known and the unknown variable that we need

② Since the j component of the system consist of unknown c, we only need to consider i component

③ Apply SUVAT to solve T

$s = ut + \frac{1}{2}at^2$

$-40 = -16t + \frac{1}{2} \times 2.4 \times t^2$

$1.2t^2 + 16t - 40 = 0$

$t = 10$ or $\frac{10}{3}$ (Reject $\frac{10}{3}$ because $T > 5$)

4c

①

Plug in T we found in 4b

Initial velocity of $P = -16i - 3j$

Acceleration of $P = 2.4i + j$

Time of P spent moved from A to B = 10

Displacement of P moved $= (4i + cj) - (44i - 10j)$

$= -40i + (c + 10j)$

we first identify the known and the unknown variable that we need

② Now we need to find c so we consider the j component

③ Apply SUVAT to find c

$s = ut + \frac{1}{2}at^2$

$c = -3 \times 10 + \frac{1}{2} \times 10^2$

$c = 20$

Question 4 continued

Question 4 continued

(Total for Question 4 is 10 marks)

5.

Consider the right-angle triangle

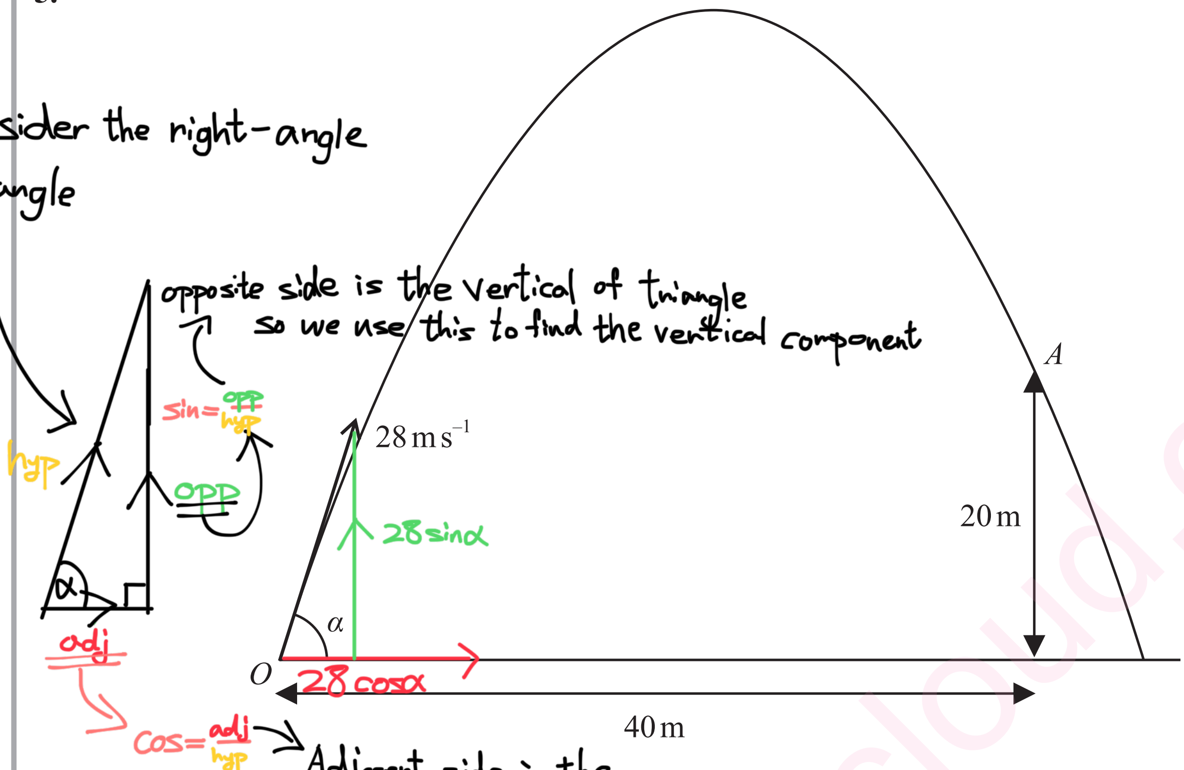


Figure 2

A small ball is projected with speed 28 m s^{-1} from a point O on horizontal ground.

After moving for T seconds, the ball passes through the point A.

The point A is 40 m horizontally and 20 m vertically from the point O, as shown in Figure 2.

The motion of the ball from O to A is modelled as that of a particle moving freely under gravity.

Given that the ball is projected at an angle α to the ground, use the model to

(a) show that $T = \dfrac{10}{7\cos\alpha}$

(2)

(b) show that $\tan^2\alpha - 4\tan\alpha + 3 = 0$

(5)

(c) find the greatest possible height, in metres, of the ball above the ground as the ball moves from O to A.

(3)

The model does not include air resistance.

(d) State one other limitation of the model.

(1)

Question 5 continued

5a

① $T = \frac{10}{7\cos\alpha}$ } As there's only 3 components in the equation, we can deduce that it is the horizontal component because it only consist velocity, time and displacement

② Use definition of velocity to show the equation

$V = \frac{S}{t}$

$T = \frac{S}{V}$

$T = \frac{40}{28\cos\alpha}$

$T = \frac{10}{7\cos\alpha}$

5b

① We have considered the horizontal component, now we consider the vertical component.

② Since the initial velocity move upward (go against gravity)

③ Initial velocity of the small ball $= 28\sin\alpha$
Acceleration of the against gravity $= -g$
Time of the small ball $= T$
Displacement of the small ball $= 20$
} we then identify the known and the unknown variable that we need

④ Then, we apply the SUVAT equation to show the equation

$S = ut + \frac{1}{2}at^2$

$20 = 28\sin\alpha\, t + \frac{1}{2} \times -g \times t^2$

Substitute 5a into the equation

$20 = 28\sin\alpha \times \frac{10}{7\cos\alpha} - 4.9 \times \left(\frac{10}{7\cos\alpha}\right)^2$

$20 = \frac{280\sin\alpha}{7\cos\alpha} - \frac{490}{49\cos^2\alpha}$

$20 = 40\tan\alpha - 10\sec^2\alpha$

$2 = 4\tan\alpha - \sec^2\alpha$

Use $\sec^2\alpha \equiv 1 + \tan^2\alpha$ identity

$2 = 4\tan\alpha - (1 + \tan^2\alpha)$

$2 = 4\tan\alpha - 1 - \tan^2\alpha$

$\tan^2\alpha - 4\tan\alpha + 3 = 0$

Question 5 continued

5c

① We need to find the angle of the projectile motion

Let $\tan\alpha = x$,

$$x^2 - 4x + 3 = 0$$

$$x = 3 \text{ or } 1$$

$$\tan\alpha = 3 \text{ or } 1$$

$\alpha \approx 71.6°$ or $45°$ (Rejected because $71.6°$ gives a greater max height)

② We consider the vertical component of the projectile motion because we are finding the height

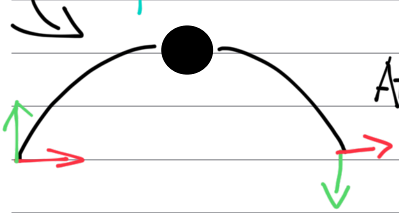
③ Initial velocity of the small ball = $28\sin\alpha$

Acceleration of the against gravity = $-g$

Final velocity of the small ball = 0

Displacement of the small ball = s

we then identify the known and the unknown variable that we need



At the maximum height, the ball will change the direction from going up to going down. Therefore, there will not be any velocity going up

④ Apply SUVAT equation to solve maximum height

$$v^2 = u^2 + 2as$$

$$0 = (28\sin\alpha)^2 - 2 \times 9.8 \times s$$

$$19.6s = 784(\sin 71.6)^2$$

$$s = 36$$

6d

Different spinning result in different velocity because it moves in different directions

Shape of the ball can affect its flight path

Question 5 continued

(Total for Question 5 is 11 marks)

6.

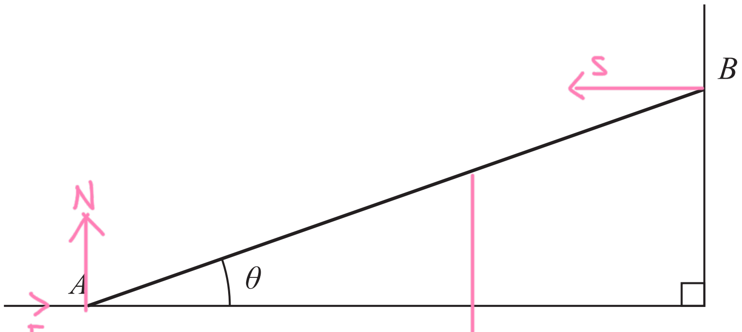


Figure 3

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

The rod is at rest in limiting equilibrium.

(a) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A. **Give a reason for your answer.**

(1)

The magnitude of the normal reaction of the wall on the rod at B is S.

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (b), (c) and (d).

(b) By taking moments about A, show that

$$S = \frac{1}{2}Mg\cot\theta$$

(3)

The coefficient of friction between the rod and the ground is μ

Given that $\tan\theta = \frac{3}{4}$

(c) find the value of μ

(5)

(d) find, in terms of M and g, the magnitude of the resultant force acting on the rod at A.

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A.

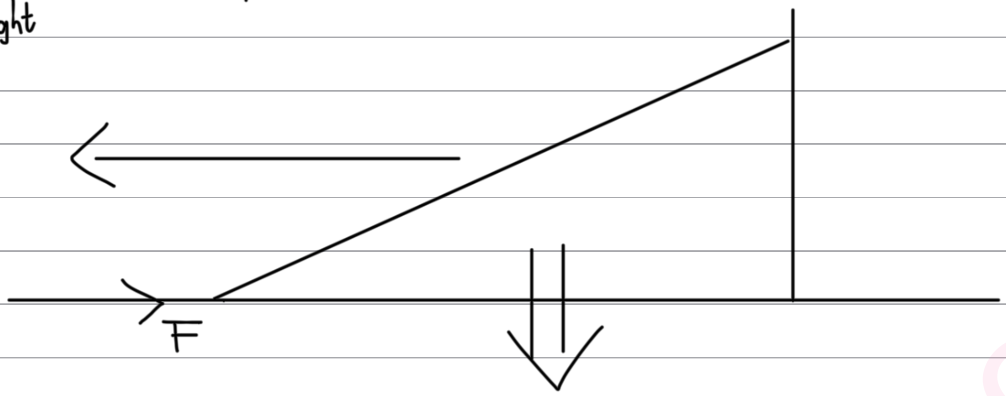
A new value for S is calculated using this new model, with $\tan\theta = \frac{3}{4}$

(e) State whether this new value for S is larger, smaller or equal to the value that S would take using the initial model. **Give a reason for your answer.**

(1)

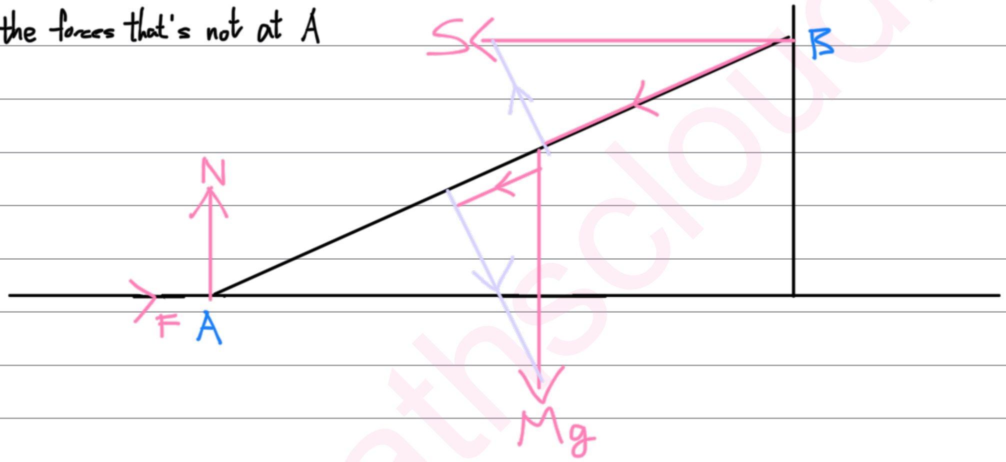
Question 6 continued

6a. The gravitational force pulls down the rod, it will move to the left. Therefore, frictional force acts to the right



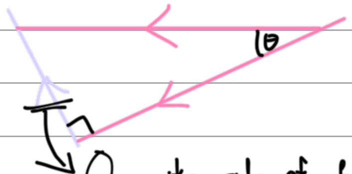
6b ① Consider the moment at A

② Identify the forces that's not at A



③ Find the forces are perpendicular to the rod

Adjecent side of force Mg



Opposite side of force S

$\uparrow F = Mg\cos\theta$ $\qquad$ $\searrow F = S\sin\theta$

④ Consider the moment at A

$$\cancel{a}\,Mg\cos\theta = 2\cancel{a}\,S\sin\theta$$

It is an uniform rod, the weight will be at the middle of 2a = a

S is at the end of the rod so it will be 2a

$$Mg\cos\theta = 2S\sin\theta$$

$$\frac{1}{2} \times \frac{Mg\cos\theta}{\sin\theta} = S$$

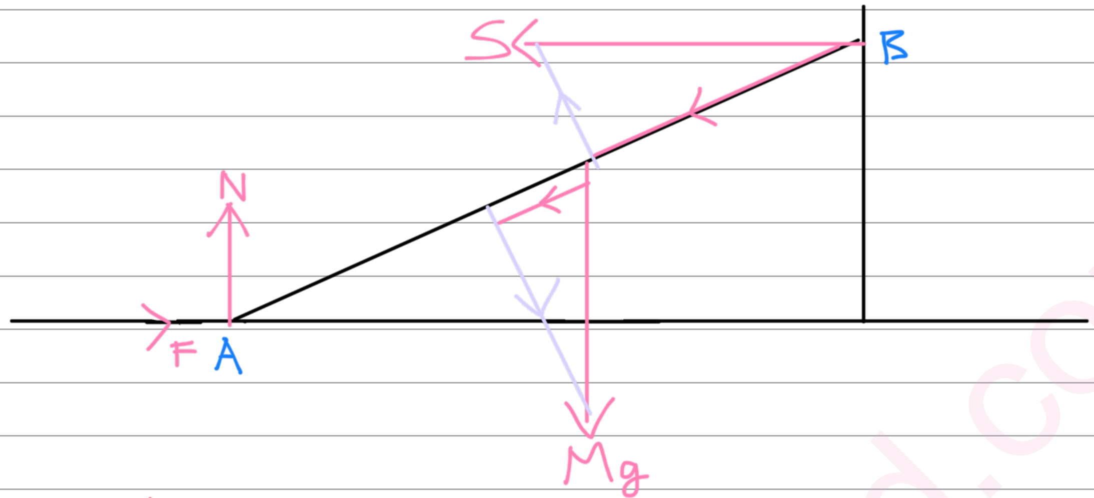
$$\frac{1}{2} Mg\cot\theta = S$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

Question 6 continued

6c



① Identify vertical forces and resolve

$N = Mg$

② Identify horizontal forces and resolve

$F = S$

③ Consider the equation $F = \mu R$

$S = \mu N$ → The Normal reaction at A

$S = \mu N$

$\frac{1}{2}Mg\cot\theta = Mg\mu$

$\frac{1}{2} \times \frac{1}{\left(\frac{3}{4}\right)} = \mu$

$\frac{1}{2} \times \frac{4}{3} = \mu$

$\frac{2}{3} = \mu$

6d ① Identify the forces acting on A

N, F

② Use Pyth. theorem to work out the resultance force

Resultant force $= \sqrt{N^2 + F^2}$

$= \sqrt{(Mg)^2 + \left(\frac{1}{2}Mg\cot\theta\right)^2}$

$= \sqrt{(Mg)^2 + \left(\frac{1}{2} \times \frac{4}{3}Mg\right)^2}$

$= \sqrt{(Mg)^2 + \left(\frac{2}{3}Mg\right)^2}$

$= \sqrt{\frac{13}{9}(Mg)^2}$

$= \frac{1}{3}Mg\sqrt{13}$

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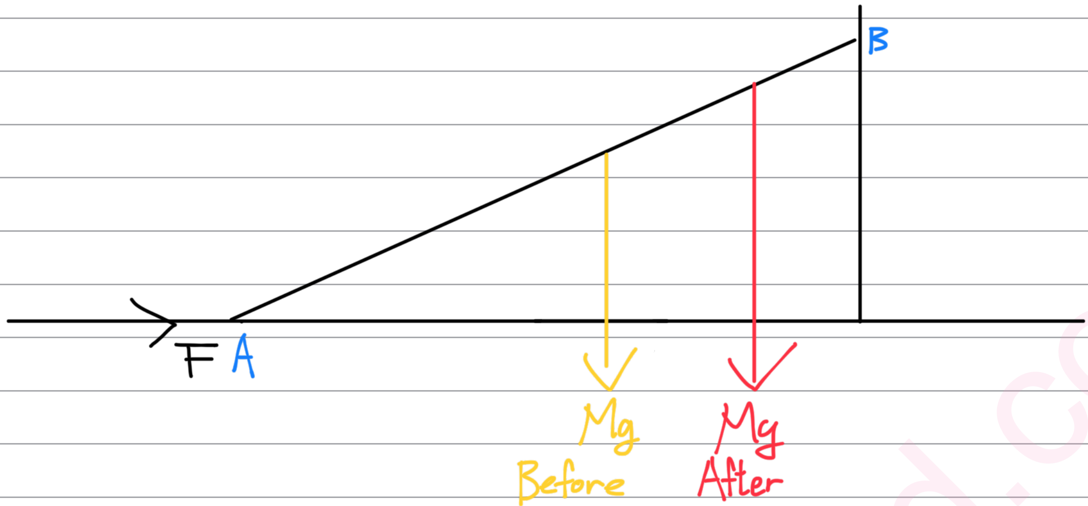
Question 6 continued

6d



① As Centre of mass moved up, distance from A to the centre of mass increase

② Moment of Mg increase

③ S increase to maintain equilibrium

Question 6 continued

(Total for Question 6 is 13 marks)

TOTAL FOR MECHANICS IS 50 MARKS